Half–Flux-Quantum Magnetoresistance Oscillations in Disordered Metal Rings

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The behavior of the magnetoresistance of single and arrays of disordered metal rings is investigated. The average localization length, L_c, which is related to the conductance, is found to oscillate with a strong half-flux-quantum harmonic at low magnetic field. The ratio of the amplitude of the full-flux oscillations versus the half-flux oscillations is shown to decrease with the number of rings. All numerical results follow a universal law where the amplitude of oscillations of L_c is found to be proportional to the square of the ratio of L_c to the perimeter of the ring. At high magnetic fields, full-flux oscillations are observed.

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The half–flux-quantum oscillations of magnetoresistance of disordered metal rings and cylinders, predicted by Al'tshuler, Aronov, and Spivak (AAS),[1] have been observed experimentally on disordered cylinders and arrays of rings by several groups.[2–5] Experiments on single rings, on the other hand, showed complicated features. Full–flux-quantum oscillations were observed by Webb *et al.*[6] on small rings. Later, Chandrasekhar *et al.*[7] observed $h/2e$ oscillations at low field and h/e oscillations at higher field on single aluminum and silver rings. Very recently both h/e and the AAS $h/2e$ oscillations were measured[8] in samples consisting of N rings connected in series. Clear evidence was found that averaging leads to a $N^{-1/2}$ dependence of the amplitude of the h/e oscillations while the amplitude of the $h/2e$ oscillations is independent of the number of rings.

The AAS effect results from the quantum corrections to conductance due to backscattering interference.[9] Theoretical works based on calculation of the transmission coefficient in one-dimensional metal rings,[10–12] however, showed that the fundamental period of oscillation at zero temperature is the full-flux quantum and that higher harmonics become dominant only at special conditions. The very important difference between the two theories is that the former employed the ensemble average. Carini, Muttalib, and Nagel[13] also showed that the existence of degeneracies and time-reversal invariance of the Hamiltonian after ensemble average could lead to $h/2e$ oscillations. It has recently been shown[14] that for a symmetric normal-metal ring, averaging of the transmission coefficient T over disorder gives oscillations with a period of $h/2e$. As the elastic scattering gets stronger, the periodicity of the magnetoresistance oscillations becomes h/e. Numerical simulation of the transmission-matrix method by Stone and Imry[15] also showed $h/2e$ oscillations when ensemble averaging was performed. The existence of uncorrelated regions and thermal smearing[15] are important sources of self-averaging. Complications due to the aperiodic fluctuations added to the magnetoresistance also exist in small samples.[16]

In this paper, we present the results of a detailed numerical simulation on a single small ring and rings connected in series. The disorder, width, magnetic field, and length dependence of the transmission coefficient, which is related to L_c, for the rings will be presented.

We simulate a small rectangular ring by a two-dimensional strip described by a nearest-neighbor tight-binding Hamiltonian

$$H=\sum_{lm}\varepsilon_{lm}\,|lm\rangle\langle lm\,|+\sum_{l'm'lm}V_{lm,l'm'}\,|lm\rangle\langle l'm'\,|. \quad (1)$$

$|lm\rangle$ denotes the site corresponding to the respective x,y coordinates. The strip boundary geometry can be set by proper choice of the site energies ε_{lm} and the hopping matrix elements $V_{lm,l'm'}$. ε_{lm} are generally taken as independent random variables uniformly distributed between $\pm W/2$ in the disordered region of the ring. As a result of our method of calculation outside the ring area, ε_{lm} are set to very large values (infinite potential barriers) so that the electrons will be moving only inside the ring area. A uniform magnetic field, normal to the ring, introducing a phase to the hopping matrix element can be taken into consideration via Peierls substitution[17]:

$$V_{lm,l'm'}=\begin{cases}1 & \text{if } m=m' \text{ and } l=l\pm 1,\\ e^{\pm i\alpha l} & \text{if } l=l' \text{ and } m'=m\pm 1,\end{cases} \quad (2)$$

where α is the number of flux quanta per unit cell, $\alpha=eB/h$, and the lattice constant is taken to be 1.

We use the recursive Green's function method to calculate[17] the localization length. Free boundary conditions are used in the transverse direction (y direction). The imaginary part of the energy is taken to be 2×10^{-4} in units of V. This is small enough that the inelastic scattering length is larger than the size of the rings studied. Since the Aharonov-Bohm effect depends only on the topology, following Stone,[15,16] we make a small hole in the middle of the strip but with much larger magnetic field in the hole than in the annulus. This way we can save on computer time and achieve a good aspect ratio (the ratio of the flux through the hole to that through the annulus), which is important in order to observe $h/2e$ oscillations. We also attach on one end a semi-infinite disordered "lead" to achieve stable values for the localization length. The geometry of the ring is the following: L is the length of the long side of the rectangular ring, M is the width of the strip, and MW is the width of the ring. In our numerical studies we always calculate the localization length which is related to the conductance by Landauer's formula[12]

$$G=(e^2/\pi\hbar)/(e^{2L/L_c}-1), \tag{3}$$

where L is the perimeter of the loop and L_c is the localization length. Throughout this paper, formula (3) will be adopted.

We organize our results as follows:

(1) *Single ring without average over disorder.*—A single ring at zero temperature does not have a self-average property.[15] For $L_c \gtrsim L$ we expect an oscillation of fundamental period h/e with a smaller $h/2e$ component. This is indeed what we observed in our simulation. For strong disorder, as can be seen in Fig. 1(a), an h/e oscillation is observed for all the widths studied. However, for some specific configuration of weak disorder, where $L_c \gg L$, a very good $h/2e$ oscillation was present as seen in Fig. 1(b). These accidental $h/2e$ oscillations are very sensitive to disorder configurations and energy and gradually transform to h/e oscillations in higher magnetic field. In most of the cases that we studied the h/e component dominates, in agreement with results on the transmission coefficient for a normal-metal ring with contacts.[10–12,14]

(2) *Single ring with ensemble average.*—In all the cases that we studied, an average over different configurations of disorder gives an $h/2e$ oscillation in the localization length and, through Eq. (3), in the conductance of a single ring. This is clearly seen in Fig. 1(c) where L_c versus the flux through the ring is plotted. Note that the $h/2e$ oscillation is the dominant one. Therefore, the ensemble averaging brings the period $h/2e$. The convergence, however, can be very slow when $L_c \gg L$. Note also in Fig. 1(c) that for strong magnetic fields, the $h/2e$ oscillation is not too dominant and the h/e oscillation starts to appear. For even stronger magnetic fields, as one can see from the inset of Fig. 1(c), there is even more structure within a full-flux period. This feature has to be investigated further.

(3) *Series-connected rings.*—$h/2e$ oscillations are seen again in this case too, when a large number of rings are connected in series. In Fig. 1(d) we show results for $N=50$ rings in series, of width $MW=4$, and disorder $W=2.0$, averaged over 100 different configurations of disorder; a perfect $h/2e$ oscillation is observed. Again in this case the dominant oscillation is the $h/2e$.

(4) *h/e vs $h/2e$ component.*—Recently, detailed experiments[8] have been performed to test the stochastic averaging of the h/e oscillations. In the experiment[8] it was clearly shown that the h/e and the $h/2e$ oscillations coexist when the ring number is small. At constant temperature, the h/e component was found to decrease with the square root of the number of loops N, while the amplitude of the $h/2e$ component was independent of the number of rings. We have performed systematic simulations to check the experimental results and predict possi-

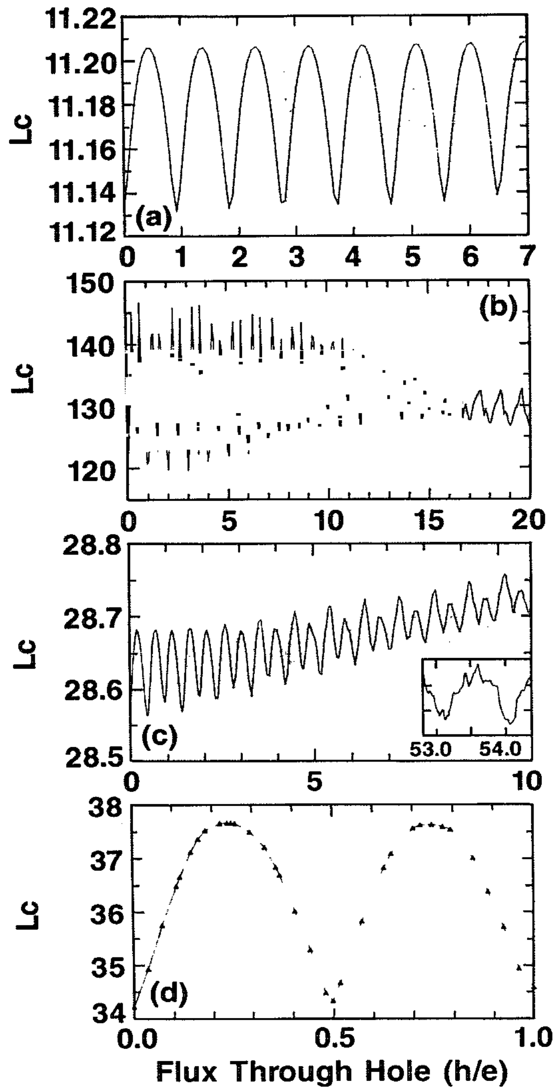

FIG. 1. Quantum oscillations of the localization length L_c, which is related to conductance G through Eq. (3), for the following cases: (a) A single ring with disorder $W=4.0$. No average over disorder is taken. The h/e oscillation is clearly observed. (b) A single ring with disorder $W=1.2$. No average over disorder is taken. The $h/2e$ oscillation is observed at small magnetic fields B and at high B gradually changes to h/e. (c) A single ring with disorder $W=2.0$. An average over 1000 disorder configurations is taken. Inset: the behavior of L_c for very strong magnetic fields. (d) A series of fifty rings with disorder $W=2.0$. An average over 100 disorder configurations is taken. In all the cases the width and length of the ring are $MW=4$ and $L=30$, respectively. Aspect ratio is 7.

ble new behaviors. We have used 100 configurations to represent the finite temperature.[15] Then we add single rings one by one in series. To extract the h/e and $h/2e$ components from the total conductance (localization length L_c) oscillation, we use an approximation which neglects higher harmonics and assume that the total conductance consists of only the h/e component and $h/2e$ component, i.e.,

$$G(\phi/\phi_0)=G_0+0.5A_1\cos(\phi/\phi_0)+0.5A_2\cos(2\phi/\phi_0), \quad (4)$$

where $\phi_0=h/e$. As can be seen from Fig. 1, the curves are relatively smooth and so the approximation in Eq. (4) is fine. Therefore, we have that $A_1=|G(\frac{1}{2})-G(0)|$ and

$$A_2=|G(0)-G(\tfrac{1}{4})+G(\tfrac{1}{2})-G(\tfrac{3}{4})|/2.$$

In Fig. 2 we plot the relative values of A_1 with respect to A_2 as a function of the ring number for two different geometries. We see that the h/e component decreases in inverse proportion to the number of rings relative to the $h/2e$ component. This seems to disagree with the experimental results[8] which claim a $1/\sqrt{N}$ decrease of the h/e component. This discrepancy might be a result of the different way of extracting the two components in our simulations and in the experiments.

(5) *Amplitude of oscillation.*—The size of the interference effects depends on the range of coherence of the diffusion electrons and is limited by other phase-breaking processes. Amont the most important are inelastic scattering and magnetic scattering. In our model, the relevant physical lengths are the perimeter of the loop, the localization length, and the magnetic field length $L_B \sim h/e(MW)B$. At low magnetic fields B, it is the ratio of L_c to L which determines the size of the oscillation. To test this idea, we numerically calculated the conductance of rings of different perimeters, different widths, and different strengths of disorder W. The results are shown in Fig. 3, where the amplitude of the oscillation for small fields B is plotted as a function of L_c/L. We indeed see that all the data of the different cases that we examined scale in a universal curve. In particular, we have that the magnitude of the oscillation of the localization length L_c is $\Delta L_c/L=0.067(L_c/L)^2$, for all the widths, perimeters L, and disorder W. Notice that as L_c/L increases, the magnitude of the conductance oscillations also increases. Our results suggest that it is possible to have conductance fluctuations[18] of values higher than e^2/h. In particular, for the single-ring geometry and $L_c/L=35$, we obtain $\Delta G=10.4e^2/h$ as the conductance fluctuation. Meanwhile, for a series of rings and $L_c/L=10$, we obtain $\Delta G=8.3e^2/h$. Stone and Imry[15] have only calculated a particular case of L_c/L and their results are in qualitative agreement with ours. In particular, from the top of Fig. 2 in Ref. 15 we have $G_{max}=3.20e^2/h$ and the magnitude of the $h/2e$ conductance oscillations is given by $\Delta G=0.05e^2/h$. From the value of G and Eq. (3) we obtain $L_c/L=4.10$, and from the ΔG value we have $\Delta L_c/L=0.05$ which is lower than our estimates. This difference is due to the fact that Stone and Imry[15] used a single ring and averaged over disorder. In Fig. 3 we plot the data for fifty rings connected in series and then averaged over disorder. The single-ring data also follow a universal law, the only difference being that $\Delta L_c/L \simeq 0.0025(L_c/L)^2$ instead of $\Delta L_c/L \simeq 0.067(L_c/L)^2$. These laws will break down when $\Delta L_c/L_c$ is of order 1. We want to mention that the data shown in Fig. 3 are the overall amplitude oscillation of L_c for small field B. For $L_c<L$, the $h/2e$ component is the dominant oscillation, while for $L_c>L$ we might have an $h/2e$ oscillation but sometimes an h/e oscillation is also present.

In conclusion, our work demonstrates unambiguously that single tight-binding rings, when averaged over disorder, do exhibit quantum interference effects, with a flux

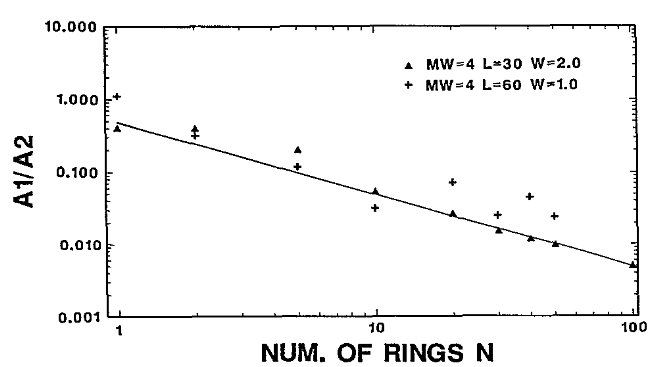

FIG. 2. The ratio A_1/A_2 vs the number of rings N on a log-log plot for different values of lengths L and disorder W. A_1 and A_2 are the amplitudes for the h/e and $h/2e$ oscillations of the conductance, respectively.

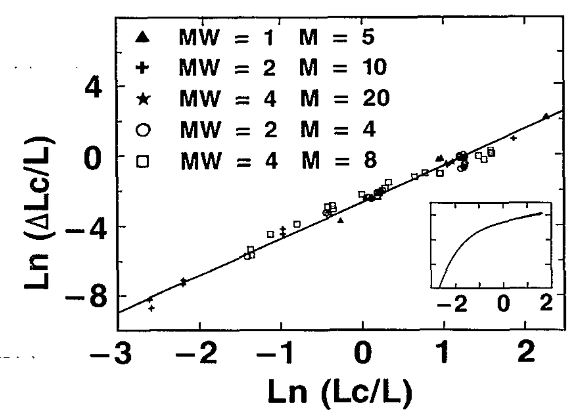

FIG. 3. Magnitude ΔL_c of the oscillation of localization length L_c vs L_c/L for different widths, lengths, and disorder. The number of rings ranges from 40 to 100. Inset: the magnitude of the conductance oscillation plotted vs L_c/L.

period $h/2e$ at low fields and with a flux period of h/e clearly visible at higher fields [see, for example, Fig. 1(c)]. This behavior has been suggested in Refs. 10–12, but it was only recently shown numerically to be correct by Refs. 14 and 18 for the transmission coefficient. This picture agrees with the experimental results. Our numerical results provide a universal law for the strength of the oscillation of the magnetoresistance. The oscillation strength only depends on the ratio of the relevant lengths, the ring perimeter, and the localization length. In particular, when $L_c \lesssim L$ the magnetoresistance is $h/2e$ periodic, but as L_c increases the h/e harmonic dominates and finally when $L_c \gg L$ the $h/2e$ harmonic might disappear completely. At the other extreme, when $L_c < L$, the $h/2e$ periodicity persists but the oscillation amplitude decreases with respect to the background value. It will be very interesting to check experimentally this universal behavior. The above phenomena have been observed for rings of width $MW=1$ as well as for rings of finite width. For the finite-width rings, the $h/2e$ oscillation is seen when the mean free path l, which is different from L_c, becomes smaller than L. Details of this behavior will be published elsewhere. Finally, by our increasing the temperature, the inelastic mean free length decreases and will eventually become smaller than L. This phase incoherence introduced by an increase of the temperature will destroy both periods of the magnetoresistance.

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